# Allan time

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**Abstract**

Stability tests based on the Allan variance method have become a standard procedure for the evaluation of the quality of radio-astronomical instrumentation. They are very simple and simulate the situation when detecting weak signals buried in large noise fluctuations. Here a practical example will be presented on the basis of the Callisto frequency agile radio spectrometer. The result will be a numerical value called Allan-time which defines the suggested period of calibration.

## 1 Introduction

All radio-astronomical measurements are affected by instabilities of the gain, the transmission function, and the internal system noise changing the absolute scale of the measured signal. To compensate for these drifts, one switches between the astronomical source and a reference signal - a known internal calibrator or a point on the blank sky - on a timescale short compared to the instabilities. Because of the overheads introduced by switching to the calibration source, the optimum strategy is not to switch as fast as possible, but only as fast as necessary to suppress the drift noise. Therefore the characteristic timescales of the instabilities have to be measured. This can be done in terms of the Allan variance [Allan D.W. (1966)], a powerful technique to determine the stability of general radioastronomical equipment, in particular for systems consisting of heterodyne receivers and spectrometer backends. The Allan variance plot can be computed from any sufficiently long time series of spectrometer dumps (light curve) taken at fixed instrumental settings, provided that the integration times for the individual dumps are small compared to all instabilities. Allan variance in general is the method of analysis of stochastic processes in a time domain. It was originally designed for the statistics of atomic frequency standards

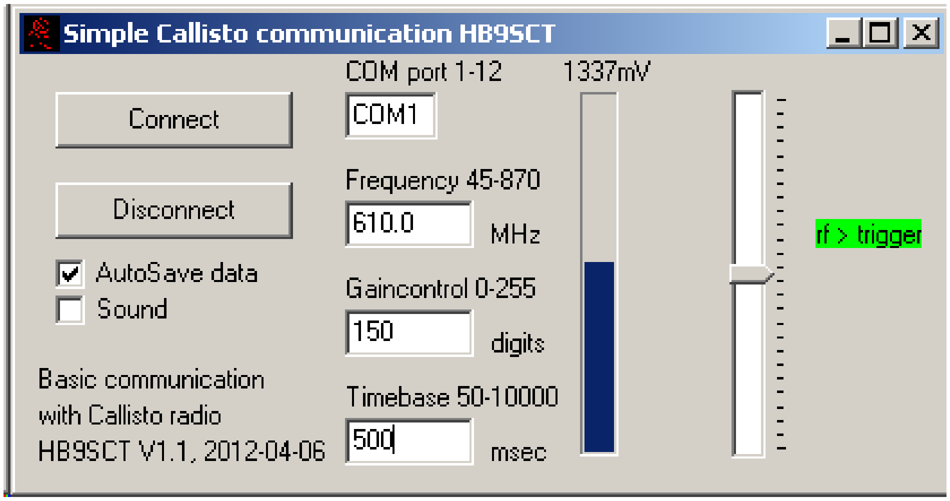


Figure 1: Setup of software tool simple.exe to control serial communication port, receiver frequency, receiver gain and sampling rate of the Callisto frequency agile radio spectrometer.

[Allan D.W. (1966)]. The main advantage of Allan variance in a comparison with power spectral density is a lower computational complexity. On the other hand, there is a problem in interpretation of some kinds of noises, which appear in the same way. However, a solution is using the modified Allan variance. Because of the problem analogy, the Allan variance method can be used for the investigation of all periodic sampling devices like digital multi-meters, frequency counters, radios, radiometers and spectrometers as well. For detailed theory, mathematical background and explanations I refer to literature below.

## 2 Theory

**The longer we average the better the result?** The answer is no. But from theoretical point of view a measuring result should get more and more precise the longer we integrate or average the incoming signal. The resolution $\sigma$ of any result is proportional to

$$\sigma \propto \frac{1}{\sqrt{N}} \tag{1}$$

where N denotes the number of measurements. If we average $N = 100$ measurements, the resolution $\sigma$ is getting 10 times better [Kraus J.D. (1980)].

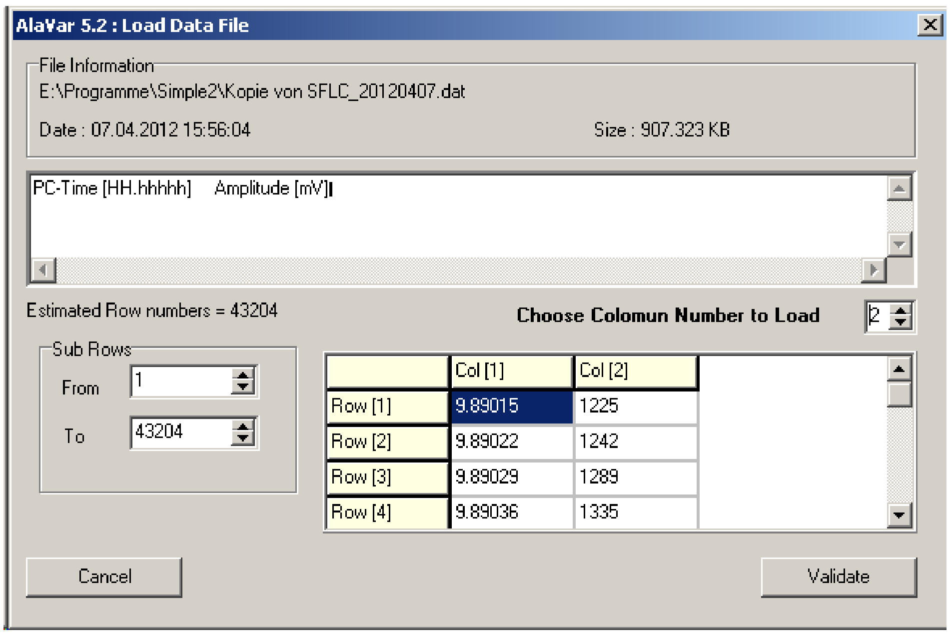


Figure 2: Reading the ASCII data-file and defining its data structure. Here column (1) contains observation time in UT and column (2) contains the amplitude of the signal to be analyzed in mV.

But this is only true if the measured signal follows purely statistical behavior (Gaussian distribution). Any nonlinearity or systematical error in the incoming signal or in the signal path (typically introduced by temperature-changes or aging processes) cannot be improved by simply averaging. To compensate for systematical errors we need a periodic calibration process. To find out the time period for calibration we need to know how long the system is stable enough without any calibration

**What is the expected rms?** Before we do the analysis with ALAVAR we should have an idea about the expected rms (root mean square) or sigma of the original signal. Again according to [Kraus J.D. (1980)] we get.

$$\sigma = \frac{\delta I}{I} = \frac{1}{\sqrt{B\ \tau}} \tag{2}$$

where I stands for any kind of intensity or power or flux or antenna temperature. Variable B stands for bandwidth which is 300 $KHz$ in case of Callisto. And $\tau$ defines the integration time which in case of Callisto is about 1 $msec$. We express $\sigma$ in dB then we get.

$$\sigma_{dB} = 10 \cdot log\left(1 + \frac{1}{\sqrt{B\ \tau}}\right) \tag{3}$$

In addition we know in case of Callisto the detector slope $g = 25.4\ mV/dB$. Knowing this we may express $\delta I_{dB}$ now in $mV$.

$$\sigma_{mV} = 10 \cdot log\left(1 + \frac{1}{\sqrt{300\ KHz \cdot 1\ msec}}\right) \cdot \frac{25.4\ mV}{1\ dB} = 6.2\ mV \tag{4}$$

In a recent measurement W. Reeve got with one of his new series Callisto $\sigma = 5.7\ mV$ while I got with my own series $\sigma = 5.8\ mV$ which both are slightly too small compared with theory. The meaning of these results is that either the bandwidth is slightly higher than 300 $KHz$ or the integration is slightly higher than 1 $msec$ or the detector slope is slightly lower than 25.4 $mV/dB$ or a combination. But the result as such proves that the low pass filtering of the detector signal is satisfactory (a measured value larger than the theoretical value, would indicate an inadequate low-pass filter).

## 3 Practical process

**Measurement setup:** Here I report about getting the Allan-time based on the Callisto frequency agile radio spectrometer. The procedure is the

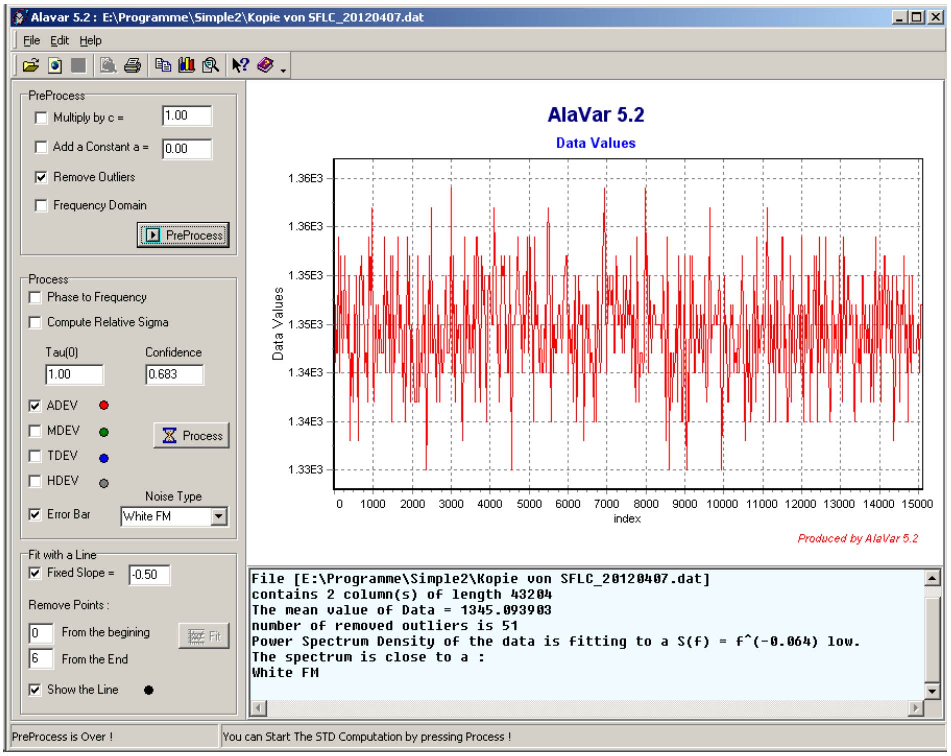


Figure 3: PreProcess of sampled data at one single frequency channel and generating light curve. The plot shows noise with rather low interference (no large spikes or gaps). The x-axis shows the sample number and the y-axis the measured voltage in millivolts.

same for any other radio, radiometer or spectrometer. The whole process has to be repeated for each frequency of interest. In my case I decided first for $f = 610.0MHz$ because it's the only frequency at my current location in the whole sky spectrum within the Callisto receiver frequency range of 45 $MHz$ and 870 $MHz$ that is more or less free from interference. In addition it is a protected frequency for radio astronomy. Another method is to measure a 50 $\Omega$ termination resistor at constant temperature instead of a defined sky-position (Cas A, CygA, Tau A, Moon, etc.) with a lot of radio frequency interference coming from the the side lobes of the antenna. First I let the whole system warm up for at least one hour such that all components have the same temperature. During the test no component of the system should be touched to avoid temperature gradient or other electromagnetic influence to the receiver system. The receiver is then set to the frequency of interest and the signal is sampled and saved in a systematic way, e.g. one measurement every $\tau = 500$ *msec*. This sampling can be done with the free tool simple.exe, which controls Callisto and collects the measured data samples. See figure 1. Any digital-multimeter with a serial or an USB-connection to a PC can be used in a similar way.

**Observation:** The observation needs to be done for at least one hour. In many cases, the observation should go on for 24 hours or even longer to capture all possible interfering inputs. The longer the better. After sampling sufficient data points (from my experience at least 30000), typically in an ASCII file, we put the signal-file into the software application ALAVAR. The application can be downloaded freely from the web (for web-link see end of this article. Install it on the hard disk of your PC. The installation-drive and -path can be selected during the installation. Start the application ALAVAR from the desktop and select the function File→Load data and select the data file. Normally the first row of the data file is interpreted as title text and is shown in the top memo-field of ALAVAR. For the "Choose number column to load" field, select 2 and then press Validate. See figure 2.

**Analysis:** If you are suffering from a lot of outliers from a **known** reason or source the you may tick the option Remove Outliers and press the button PreProcess. But this action needs to be mentioned in the final report, otherwise the report will look too good. If you know that there is no interference coming into the receiver then you may skip the function Remove Outliers (but you still must press PreProcess). Then we immediately get the light curve plot as shown in figure 3. The lightcurve plot should appear random

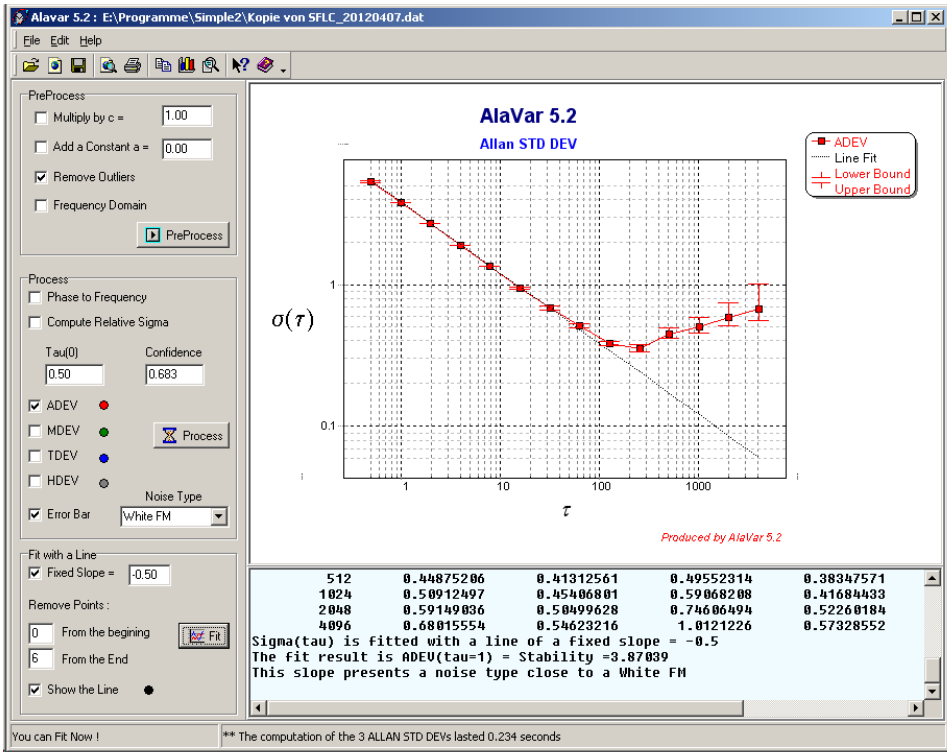


Figure 4: ALLAN-variance plot for Callisto at $f = 610.0\ MHz$. The x-axis shows the integration time $\tau$ expressed in seconds over an observation time of 6 hours and 3 minutes, the y-axis the variance $\sigma$ of the signal, both in logarithmic scale.

with no periodic indications. As the next step we should not forget to enter the sampling time Tau, in my case 0.5 sec. Then tick ADEV (overlapping allan standard deviation in red) and Error Bar but leave MDEV (modified Allan standard deviation), TDEV (Allan time standard deviation) and HDEV (overlapping Hadamard variance) un-ticked. The tool ALAVAR offers different statistical methods for analysis. As amateur radio astronomers we stick on ADEV (overlapping allan standard deviation in red) because it is the most common statistical method, although there is no large difference between it and the other methods. ADEV is fine for our purposes, which is to get an idea about stability in time of the receiver. For more information, go to the ALAMATH-website of ALAVAR (link below). Then press the button Process. One immediately gets the Allan-plot as shown in figure 4. Now set the Fixed Slope to -0.50 (purely theory) and press button Fit. Then one should get the theoretical line fit plot for purely statistical noise. In practice the measured situation (red) will differ from theory (gray). Therefore one needs to cancel several points from the end of the plot, in my case 6 points, such that the measured plot follows in parallel as long as possible the theorectical line fit trace. Press again the button Fit.

## 4 Result/Conclusion

In my example based on Callisto the Allan-time is in the order of 250 seconds, as indicated by the minimum in the red trace in figure 4. That means we can measure and integrate up to this time and we still get results with improving precision. If we integrate longer than the Allan-time (250 *msec* for my receiver) the result will get worse, the red plot increases again. Therefore, in practice we need to recalibrate the whole system about every 4 minutes to compensate for systematical errors. Appropriate calibration methods and a calibration unit will be presented in a future article.

## 5 Relevant internet addresses for download, further reading and study

- CALLISTO→ `http://www.e-callisto.org`
- ALAVAR→ `http://www.alamath.com/`
- Allantime→ `http://en.wikipedia.org/wiki/Allan_variance`

- NIST Technical Note 1337, Characterization of Clocks and Oscillators `tf.nist.gov/general/pdf/868.pdf`
- Allan's paper at NIST `tf.nist.gov/timefreq/general/pdf/7.pdf`

## References


[Allan D.W. (1966)] Statistics of atomic frequency standards. Proceedings of the IEEE 54, 2 (feb. 1966), 221230.

[Kraus J.D. (1980)] Radio astronomy, Cygnus Quasar Books.